

Space – time symmetry of noncommutative field theory

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Abstract

We consider the deformed Poincare group describing the space-time symmetry of noncommutative field theory. It is shown how the deformed symmetry is related to the explicit symmetry breaking.

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1. Introduction

In the two recent interesting papers [1, 2] (cf. also [3]) Chaichian et al. proposed a new interpretation of the symmetry of noncommutative space-time defined by the commutation relations:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu} \quad (1)$$

where $\Theta^{\mu\nu}$ is a constant antisymmetric matrix.

According to the standard wisdom the relation (1) break Lorentz symmetry down to the stability subgroup of $\Theta^{\mu\nu}$. Therefore, the exact symmetry of noncommutative space-time is the semidirect product of the latter and translation group. In spite of that all fundamental issues of the noncommutative quantum field theory (NCQFT) [4, 5] are discussed in fully covariant approach using the representations of the Poincare group. Moreover, when trying to base the theory on the stability subgroup of $\Theta^{\mu\nu}$ one is at once faced with the question why the multiplets of stability subgroup are organised in such a way as to form the complete multiplets of full Lorentz/Poincare group.

The way out of this dilemma proposed by Chaichian et al. consists in the following. One poses the question whether the noncommutative space-time admits as large symmetry as its commutative counterpart provided the symmetry is understood in the wider sense of quantum groups theory. If this is the case one can try to use this generalised symmetry as the substitute of Poincare group.

Chaichian et al. have shown that this scenario works quite well. Actually, they constructed the infinitesimal version of deformed Poincare symmetry which appeared to be the standard Poincare algebra equipped, however, with the modified (twisted) coproduct. They gave also a number of convincing arguments that the generalised symmetry is as efficient as the Poincare symmetry in the commutative case.

On the other hand one can consider NCQFT from different point of view. It emerges, in particular circumstances, as a specific limit of fully symmetric theory. Therefore, it is not simply an example of the theory with some prescribed symmetry group which, accidentally, appears to be a subgroup of the Poincare group. On the contrary, it is an example of what is called explicit symmetry breaking.

Explicit symmetry breaking is a well-known notion. Generically such a symmetry pattern can be described as follows. All dynamical variables as well as all parameters entering have well-defined transformation properties under some group G . However, G itself is not good candidate for the symmetry group except that all *parameters* are invariant under G . If this is not the case only the stability subgroup $H \subset G$ of all parameters can serve as a symmetry group.

On the lagrangian level all that means that the Lagrangian is invariant under G -transformation of all dynamical variables as well as all parameters. However, symmetry demands more, namely the invariance under the transformation of dynamical variables provided the values of parameters are kept fixed. Consequently symmetry transformations must belong to H .

When the symmetry group H emerges from explicit breaking of the larger group G one can say more about the structure of the system than it follows by merely viewing H as a symmetry group. Some aspects can be explained in terms of the exact symmetry H while in order to understand other properties one has to appeal to the initial group G . On the Lagrangian level one expects, for example, the currents related to the generators of H to

be conserved while those related to generators of G/H not; however, the invariance of the lagrangian under simultaneous G -transformations of dynamical variables and parameters restricts the admissible form of current divergencies. In fact, the (nonvanishing) divergencies can be calculated from the variation of the Lagrangian when the parameters vary under the action of G .

The point we want to make in the present note is that in some cases the explicit symmetry breaking $G \downarrow H$ can be described in terms of deformed G -symmetry, the deformation parameters being related to the parameters appearing in those terms of the lagrangian which are responsible for symmetry breaking.. The quantum symmetry, being a more general notion, should impose less severe restrictions on basic characteristic of the system. On the other hand, deformed G (containing undeformed H as a substructure) is more than H itself. Therefore we cannot expect quantum G to imply conservation of all currents but we can expect it to impose some restrictions on current divergencies; in fact, they should be the same as those implied by explicit $G \downarrow H$ breaking. We show that this scheme really works in the case of field theory on noncommutative space-time. To this end we consider the global counterpart of infinitesimal group of Chaichian et al. and discuss the Noether identities following from generalised symmetry. They are the same as the ones implied by explicit symmetry breaking and the divergencies of the corresponding currents can be related equivalently either to the variation of the Lagrangian under the change of the symmetry breaking parameters or deformation of the symmetry group.

2. Global Θ -Poincare symmetry

The global counterpart of Chaichian et al. algebra has been found few years ago [6, 7] and rediscovered recently [8].

The corresponding *-Hopf algebra is generated by Hermitean elements $\hat{\Lambda}^\mu{}_\nu$, \hat{a}^μ obeying

$$\begin{aligned}
[\hat{a}^\mu, \hat{a}^\nu] &= -i\Theta^{\rho\sigma}(\hat{\Lambda}^\mu{}_\rho\hat{\Lambda}^\nu{}_\sigma - \delta^\mu{}_\rho\delta^\nu{}_\sigma) \\
[\hat{\Lambda}^\mu{}_\nu, \bullet] &= 0 \\
\Delta(\hat{a}^\mu) &= \hat{\Lambda}^\mu{}_\nu \otimes \hat{a}^\nu + \hat{a}^\mu \otimes \mathbf{1} \\
\Delta(\hat{\Lambda}^\mu{}_\nu) &= \hat{\Lambda}^\mu{}_\alpha \otimes \hat{\Lambda}^\alpha{}_\nu \\
S(\hat{a}^\mu) &= -\hat{\Lambda}^\mu{}_\nu \hat{a}^\nu \\
S(\hat{\Lambda}^\mu{}_\nu) &= \hat{\Lambda}^\mu{}_\nu \\
\epsilon(\hat{a}^\mu) &= 0 \\
\epsilon(\hat{\Lambda}^\mu{}_\nu) &= \delta^\mu{}_\nu
\end{aligned} \tag{2}$$

The relations (2) define a deformation P_Θ of Poincare group (Θ -Poincare group). P_Θ acts on noncommutative space-time in the standard way:

$$\hat{x}^\mu \rightarrow \hat{\Lambda}^\mu{}_\nu \otimes \hat{x}^\nu + \hat{a}^\mu \otimes \mathbf{1} \tag{3}$$

Let us note that the stability subgroup of $\Theta^{\mu\nu}$ is a classical group.

The noncommutative algebraic structure can be encoded in commutative framework via Weyl-Moyal correspondence. Let M_Θ be the algebra spanned by the generators \hat{x}^μ (Θ -Minkowski space-time). Then $P_\Theta \otimes M_\Theta$ has a very simple structure because Lorentz elements $\hat{\Lambda}^\mu{}_\nu$ belong to its center and can be viewed as commutative parameters. Writing eq.(2) in the form $[\hat{a}^\mu, \hat{a}^\nu] = i\tilde{\Theta}^{\mu\nu}$ with $\tilde{\Theta}^{\mu\nu} = -\Theta^{\rho\sigma}(\hat{\Lambda}^\mu{}_\rho\hat{\Lambda}^\nu{}_\sigma - \delta^\mu{}_\rho\delta^\nu{}_\sigma)$ one can use Weyl-Moyal

correspondence for the algebra spanned by \hat{x}^μ and \hat{a}^μ . As a result we obtain the following star product (see [9] for the idea to use the star product for the algebra of functions depending on group elements):

$$(f * g)(\Lambda, a, x) \equiv f(\Lambda, a, x) \exp \left[\frac{i}{2} \left(\Theta^{\mu\nu} \frac{\overleftarrow{\partial}}{\partial x^\mu} \frac{\overrightarrow{\partial}}{\partial x^\nu} - \Theta^{\rho\sigma} (\Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma - \delta_\rho^\mu \delta_\sigma^\nu) \frac{\overleftarrow{\partial}}{\partial a^\mu} \frac{\overrightarrow{\partial}}{\partial a^\nu} \right) \right] g(\Lambda, a, x) \quad (4)$$

for any two functions f, g of *classical* variables $\Lambda^\mu{}_\nu, x^\mu, a^\mu$.

The above star product is by construction associative. For f, g depending on x^μ only one obtains standard Moyal product while for the functions on classical Poincare group manifold our product encodes the algebraic structure defined by first two eqs.(2).

The crucial property of our star product is that it commutes with the action of the Poincare group:

$$f(x) * g(x)|_{x \rightarrow \Lambda x + a} = f(\Lambda x + a) * g(\Lambda x + a) \quad (5)$$

On the LHS one takes first the star product (the standard Moyal one) and then applies Poincare transformation while on the RHS Poincare transformation is taken before applying the star product (4).

Eq.(5) can be easily proven by noting that:

$$\begin{aligned} \frac{\partial f(\Lambda x + a)}{\partial x^\mu} &= \Lambda^\nu{}_\mu \frac{\partial f(x)}{\partial x^\nu} \Big|_{x \rightarrow \Lambda x + a} \\ \frac{\partial f(\Lambda x + a)}{\partial a^\mu} &= \frac{\partial f(x)}{\partial x^\mu} \Big|_{x \rightarrow \Lambda x + a} \end{aligned} \quad (6)$$

It is the global counterpart of the infinitesimal form obtained by Chaichian et al. (see [2] esp. Appendix). This local form can be derived as follows. One defines the Lie algebra generators by standard duality rules, which in the case of Poincare group read ($\Delta^\mu{}_\nu \equiv \Lambda^\mu{}_\nu - \delta^\mu{}_\nu$), (cf.[10]):

$$\begin{aligned} < P_\mu, a^{\nu_1} \dots a^{\nu_m} \Delta^{\rho_1}{}_{\sigma_1} \dots \Delta^{\rho_n}{}_{\sigma_n} > &= i \delta_{m1} \delta_{n0} \delta_\mu{}^{\nu_1} \\ < M_{\alpha\beta}, a^{\nu_1} \dots a^{\nu_m} \Delta^{\rho_1}{}_{\sigma_1} \dots \Delta^{\rho_n}{}_{\sigma_n} > &= i \delta_{m0} \delta_{n1} (\delta_\alpha^{\rho_1} g_{\beta\sigma_1} - \delta_\beta^{\rho_1} g_{\alpha\sigma_1}) \end{aligned} \quad (7)$$

The action of the generators is defined in the standard way [11]: let g be an element of Hopf algebra, χ - an element of dual Hopf algebra and $gx = \sum_i g_i \otimes x_i$ - the group action; then the action of dual algebra is defined by $\tilde{\chi}x = \sum_i < \chi, g_i > x_i$. Using this together with eqs. (4), (5) and (7) one easily finds

$$\begin{aligned} \tilde{P}_\mu f &= i \partial_\mu f \\ \tilde{M}_{\alpha\beta} f &= -i(x_\alpha \partial_\beta - x_\beta \partial_\alpha) f \\ \tilde{P}_\mu (f * g) &= (\tilde{P}_\mu f) * g + f * (\tilde{P}_\mu g) \\ \tilde{M}_{\alpha\beta} (f * g) &= (\tilde{M}_{\alpha\beta} f) * g + f * (\tilde{M}_{\alpha\beta} g) + \frac{1}{2} \Theta^{\mu\nu} \left(\eta_{\mu\alpha} (\tilde{P}_\beta f) * (\tilde{P}_\nu g) - \right. \\ &\quad \left. \eta_{\mu\beta} (\tilde{P}_\alpha f) * (\tilde{P}_\nu g) + \eta_{\nu\alpha} (\tilde{P}_\mu f) * (\tilde{P}_\beta g) - \eta_{\nu\beta} (\tilde{P}_\mu f) * (\tilde{P}_\alpha g) \right) \end{aligned} \quad (8)$$

The last of eq.(8) coincides with eq.(2.8) of Ref.[1] (see also [12]). This concludes our discussion of the global Θ -Poincare symmetry. Other details concerning the geometry of P_Θ will be given elsewhere.

3. Θ -Poincare symmetry vs. explicit symmetry

Let us now consider the noncommutative field theory in the Lagrangian formalism. Assume for simplicity that we are dealing with scalar fields only; the action of Poincare group is given by:

$$\Phi(x) \rightarrow \Phi(\Lambda x + a) \quad (9)$$

which differs from the standard action ($\Phi(x) \rightarrow \Phi(\Lambda^{-1}(x - a))$) but this is irrelevant for our purposes.

The general form of the Lagrangian reads:

$$L(x) = L(\Phi(x), \partial_\mu \Phi(x))_* \quad (10)$$

where the star on the right-hand side means that all products are the star ones (although some of them can be replaced by normal products when L is integrated to yield the action).

$L(x)$ can be viewed as a standard Lagrangian although depending on derivatives of arbitrary orders:

$$L(x) = \bar{L} \left(\Phi(x), \partial_\mu \Phi(x), \partial_\mu \partial_\nu \Phi(x), \dots; \Theta^{\alpha\beta} \right) \quad (11)$$

Here we have written out explicitly its dependence on the parameters $\Theta^{\alpha\beta}$.

Now, using (11) as a starting point one can derive Noether's identities. It is slightly complicated due to the appearance of higher-order derivatives but proceeds along the standard way. One writes out the condition of invariance of the Lagrangian under a given transformation (Lorentz one in the case under consideration) and attempts to rearrange it as to obtain the Euler-Lagrange expressions plus total divergence of something (the corresponding current). In our case the initial identity contains an additional term $\delta\Theta^{\mu\nu} \frac{\partial \bar{L}}{\partial \Theta^{\mu\nu}}$ because \bar{L} is Lorentz invariant only provided $\Theta^{\mu\nu}$ transforms as an antisymmetric tensor of second rank. This term gives rise to the nonconservation of the four dimensional angular momentum. Obviously the divergence of the relevant current is defined only up to the terms proportional to Euler-Lagrange expressions and the terms of the form $\partial_\mu X^\mu$, X^μ being an arbitrary four-vector constructed out of the fields and their derivatives. In particular if it can be written as a sum of such terms the corresponding current can be redefined to yield the conserved one.

The term $\delta\Theta^{\mu\nu} \frac{\partial \bar{L}}{\partial \Theta^{\mu\nu}}$ can be easily calculated by noting that $\Theta^{\mu\nu}$ enters $L(x)$ only through the star product. Under the variation $\Theta^{\mu\nu} \rightarrow \Theta^{\mu\nu} + \delta\Theta^{\mu\nu}$ the star product transforms as follows:

$$f * g \rightarrow f * g + \frac{i}{2} \delta\Theta^{\mu\nu} \partial_\mu f * \partial_\nu g \quad (12)$$

Let us consider the Lorentz transformation in the $(\alpha\beta)$ -plane, $\Lambda^\mu_\nu = (\exp(\frac{\omega}{2} M_{\alpha\beta}))^\mu_\nu$ with $(M_{\alpha\beta})^\mu_\nu \equiv \delta^\mu_\alpha g_{\beta\nu} - \delta^\mu_\beta g_{\alpha\nu}$ being the corresponding generator in the defining representation; infinitesimally, $\Lambda^\mu_\nu \simeq \delta^\mu_\nu + \frac{\omega}{2} (\delta^\mu_\alpha g_{\beta\nu} - \delta^\mu_\beta g_{\alpha\nu})$. Inserting into eq.(12) the form of $\delta\Theta^{\mu\nu}$ induced by this transformation:

$$\begin{aligned} \delta\Theta_{(\alpha\beta)}^{\mu\nu} = & \frac{\omega}{2} (\delta_\alpha^\mu g_{\beta\nu} - \delta_\beta^\mu g_{\alpha\nu}) \Theta^{\rho\nu} + \\ & + \frac{\omega}{2} (\delta_\alpha^\nu g_{\beta\rho} - \delta_\beta^\nu g_{\alpha\rho}) \Theta^{\mu\rho} \end{aligned} \quad (13)$$

one gets ($\Delta_{\alpha\beta}^{\mu} \equiv \Theta_{\alpha}^{\mu}\partial_{\beta} - \Theta_{\beta}^{\mu}\partial_{\alpha}$):

$$f * g \rightarrow f * g + \frac{i\omega}{4}\Delta_{\beta\alpha}^{\nu}f * \partial_{\nu}g - \frac{i\omega}{4}\partial_{\mu}f * \Delta_{\beta\alpha}^{\mu}g \quad (14)$$

Note the similarity between eq.(14) and the last eq.(8).

If f and/or g are themselves the star products of other factors one has to apply (14) successively keeping only terms of first order in ω . As a result one obtains for the variation of the product of arbitrary number of factors the sum of terms corresponding to all pairs of factors:

$$\begin{aligned} \delta(f_1 * f_2 * \dots * f_n) = \\ \frac{i\omega}{4}(\Delta_{\beta\alpha}^{\nu\mu} - \Delta_{\beta\alpha}^{\mu\nu})(\sum_{k < l} f_1 * \dots * \partial_{\mu}f_k * \dots * \partial_{\nu}f_l * \dots * f_n) \end{aligned} \quad (15)$$

where $\Delta_{\alpha\beta}^{\mu\nu} \equiv \Theta_{\alpha}^{\mu}\delta_{\beta}^{\nu} - \Theta_{\beta}^{\nu}\delta_{\alpha}^{\mu}$.

Eq.(15) allows us to calculate the divergence of Lorentz current for any theory. Of course, as we have mentioned above, the resulting expression can be further modified by adding/subtracting admissible terms.

Let us consider now the Noether identities within Θ -Poincare symmetry formalism. In the nondeformed case the equivalent way of deriving the Noether identities is to compute the action of Lorentz generator on L . Adopting this method in Θ -Poincare case we apply $i\tilde{M}_{\alpha\beta}$ to L and use eq.(8). Now, L is a sum (finite or infinite) of star-product monomials in fields and their derivatives so one has only to compute the action of $i\tilde{M}_{\alpha\beta}$ on such monomials. According to standard arguments [11] this amounts to find $\dots(id \otimes id \otimes \Delta) \circ (id \otimes \Delta) \circ \Delta \tilde{M}_{\alpha\beta}$. Using eq.(8) we finally arrive at the expression of the form:

$$\begin{aligned} i\tilde{M}_{\alpha\beta}(f_1 * f_2 * \dots * f_n) = \sum_{k=1}^n f_1 * \dots * i\tilde{M}_{\alpha\beta}f_k * \dots * f_n + \\ \frac{i}{2}\sum_{k < l}\left(f_1 * \dots * (\Theta_{\alpha}^{\nu}\tilde{P}_{\beta} - \Theta_{\beta}^{\nu}\tilde{P}_{\alpha})f_k * \dots * P_{\nu}f_l * \dots * f_n + \right. \\ \left.- f_1 * \dots * P_{\nu}f_k * \dots * (\Theta_{\alpha}^{\nu}\tilde{P}_{\beta} - \Theta_{\beta}^{\nu}\tilde{P}_{\alpha})f_l * \dots * f_n\right) \end{aligned} \quad (16)$$

which is equivalent to eq.(15).

When calculating $i\tilde{M}_{\alpha\beta}L$ we get two types of terms generated by two sums on the right-hand side of eq.(16). First there will be a sum of terms corresponding to various monomials entering the Lagrangian with the operator $i\tilde{M}_{\alpha\beta}$ inserted in all possible ways. This is one-to-one correspondence with the commutative case except that the proper ordering must be observed due to noncommutativity of the star product. In order to use field equations one has to rearrange the ordering as to put the terms with $i\tilde{M}_{\alpha\beta}$ left- or rightmost. This yield some star commutators; however, it is known that such commutators can be written as total divergence and included in the divergence of the angular momentum current. We conclude that the angular momentum tensor can be obtained by properly ordering its commutative counterpart, replacing ordinary products by the star ones and adding a number of well-defined expressions following from star-commutators which appear due to reordering.

The second sum on the right-hand side of eq.(16) gives rise to the divergence of the angular-momentum current. It is exactly the same term as the one obtained from explicit symmetry breaking.

Let us conclude with a simple example Φ^3 -theory (see Refs. [13]÷[16] for similar examples):

$$L = \frac{1}{2} \left(\partial_\mu \Phi * \partial^\mu \Phi - m^2 \Phi * \Phi \right) - \frac{\lambda}{3!} \Phi_*^3 \quad (17)$$

The action $S = \int d^4x L$ is invariant under translations; applying standard Noether algorithm (which is equivalent to computing $\partial_\mu L$) one arrives at the following identity:

$$\begin{aligned} \partial_\mu \left(\frac{1}{2} (\partial_\alpha \Phi * \partial^\mu \Phi + \partial^\mu \Phi * \partial_\alpha \Phi) - \delta_\alpha^\mu L \right) - \frac{1}{2} \left((\partial^2 + m^2) \Phi + \frac{\lambda}{2!} \Phi_*^2 \right) * \partial_\alpha \Phi + \\ - \frac{1}{2} \partial_\alpha \Phi * \left((\partial^2 + m^2) \Phi + \frac{\lambda}{2!} \Phi_*^2 \right) - \frac{\lambda}{2 \cdot 3!} [\Phi, [\partial_\alpha \Phi, \Phi]_*]_* = 0 \end{aligned} \quad (18)$$

Now the \star -commutator is a total divergence:

$$-\frac{\lambda}{2 \cdot 3!} [\Phi, [\partial_\alpha \Phi, \Phi]_*]_* \equiv \partial_\mu Y_\alpha^\mu \quad (19)$$

Therefore, the "corrected" energy-momentum tensor:

$$T_{*\alpha}^\mu \equiv \frac{1}{2} (\partial_\alpha \Phi * \partial^\mu \Phi + \partial^\mu \Phi * \partial_\alpha \Phi) - \delta_\alpha^\mu L + Y_\alpha^\mu \quad (20)$$

is conserved, as it should be since the theory is translationally invariant.

On the other hand, proceeding as described above and making some rearrangement of terms we find the following identity for Lorentz transformations:

$$\begin{aligned} \partial_\mu \left(x_\alpha \left(\frac{1}{2} (\partial_\beta \Phi * \partial^\mu \Phi + \partial^\mu \Phi * \partial_\beta \Phi) - \delta_\beta^\mu L \right) + \right. \\ \left. - x_\beta \left(\frac{1}{2} (\partial_\alpha \Phi * \partial^\mu \Phi + \partial^\mu \Phi * \partial_\alpha \Phi) - \delta_\alpha^\mu L \right) - \frac{\lambda}{2 \cdot 3!} \partial_\mu^{-1} [[\Phi, i\tilde{M}_{\alpha\beta}\Phi]_*, \Phi]_* \right) + \\ - \frac{1}{2} \left((\partial^2 + m^2) \Phi + \frac{\lambda}{2!} \Phi_*^2 \right) * i\tilde{M}_{\alpha\beta} \Phi - \frac{1}{2} i\tilde{M}_{\alpha\beta} \Phi * \left((\partial^2 + m^2) \Phi + \frac{\lambda}{2!} \Phi_*^2 \right) + \\ + \frac{i}{4} \Delta_{\alpha\beta}^\nu \Phi * \partial_\nu \left((\partial^2 + m^2) \Phi + \frac{\lambda}{2!} \Phi_*^2 \right) + \frac{i}{4} \partial_\nu \left((\partial^2 + m^2) \Phi + \frac{\lambda}{2!} \Phi_*^2 \right) * \Delta_{\alpha\beta}^\nu \Phi + \\ - \frac{\lambda}{3!} \left(\frac{i}{2} (\partial_\nu \Phi * \Delta_{\alpha\beta}^\nu \Phi * \Phi - \Phi * \Delta_{\alpha\beta}^\nu \Phi * \partial_\nu \Phi) + \frac{i}{4} [\Delta_{\alpha\beta}^\nu \Phi, \{\Phi, \partial_\nu \Phi\}_*]_* \right) = 0 \end{aligned} \quad (21)$$

One obtains the same result by computing the divergence of $x_\alpha T_{*\beta}^\mu - x_\beta T_{*\alpha}^\mu$ using eq.(20).

4. Concluding remarks

We have shown that Θ -Poincare group defined in Refs. [7], [8] provides the space-time symmetry group for noncommutative field theory. It gives rise to proper Noether identities modified as compared with the commutative case by terms following from the modification of algebraic (in global formulation) or coalgebraic (in infinitesimal version) structure. In the case under consideration we arrive at the clear understanding of the meaning of deformed symmetry. It emerges due to the fact that we are considering the theory with explicit symmetry breaking. This means that, although the dynamics (lagrangian) is not invariant under

the full Poincare group, all dynamical variables as well as parameters entering have well-defined transformation properties. The symmetry breaking manifests itself in the form of current nonconservation which is computed using the invariance of the lagrangian under simultaneous transformations of dynamical variables and parameters. These properties can be summarized as resulting from deformed Poincare symmetry. In particular, the deformation is responsible for the nonconservation of some currents in spite of the fact that the deformed symmetry can be viewed as the exact one.

We have considered very special example of such an equivalence scheme. The symmetry breaking emerges here through the modification of the product of basic dynamical variables. One can ask whether there are other examples of the complementarity between explicit symmetry breaking picture and the one based on deformed exact symmetry and whether such a deformation is always given by a twist.

Finally, we note that the problem of symmetries of noncommutative theories with Galilean invariance has been considered in [17], [18].

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